The Analytic Form of the Forecast Function

Beyond the reach of the starting values, the forecast function can be represented by a homogeneous difference equation. The unit roots can be incorporated within the analytic solution of the difference equation. In the long run, the unit roots dominate the solution.

In general, if d of the roots are unity, then the general solution will comprise a polynomial in t of order $d-1$.

Example. For an example of the analytic form of the forecast function, we may consider the Integrated Autoregressive (IAR) Process defined by

$$\big\{1-(1+\phi)L+\phi L^2\big\}y(t)=\varepsilon(t), \tag{30}$$

wherein $\phi \in (0,1)$. The roots of the auxiliary equation $z^2-(1+\phi)z+\phi=0$ are $z=1$ and $z=\phi$. The solution of the homogeneous difference equation

$$\big\{1-(1+\phi)L+\phi L^2\big\}\hat{y}(t+h|t)=0, \tag{31}$$

which defines the forecast function, is

$$\hat{y}(t+h|t)=c_1+c_2\phi^h, \tag{32}$$

where c_1 and c_2 are constants which reflect the initial conditions. These constants are found by solving the equations

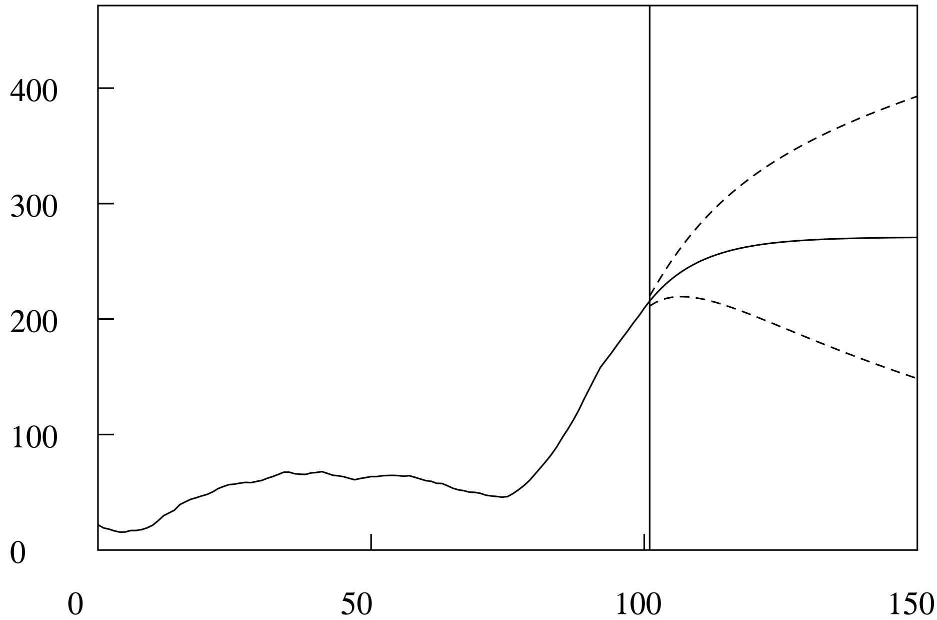
$$\begin{aligned} y_{t-1}&=c_1+c_2\phi^{-1},\\ y_t&=c_1+c_2. \end{aligned} \tag{33}$$

The solutions are

$$c_1=\frac{y_t-\phi y_{t-1}}{1-\phi} \qquad \text{and} \qquad c_2=\frac{\phi}{\phi-1}(y_t-y_{t-1}). \tag{34}$$

The long-term forecast is $\bar{y}=c_1$ which is the asymptote to which the forecasts tend as the lead period h increases.

The figure overleaf shows the trajectory of an IAR process together with the corresponding forecast function and its confidence bounds.



The sample trajectory and the forecast function of an integrated autoregressive process $(1 - 0.9L)\nabla y(t) = \varepsilon(t)$.