Forecasting an Integated Autoregressive Process

For an example of the analytic form of the forecast function, we may consider the Integrated Autoregressive (IAR) Process defined by

$$\{I-(1+\phi)L+\phi L^2\}y(t)=\varepsilon(t), \tag{57}$$

wherein $\phi\in(0,1)$. The roots of the auxiliary equation $z^2-(1+\phi)z+\phi=0$ are $z=1$ and $z=\phi$. The solution of the homogeneous difference equation

$$\{I-(1+\phi)L+\phi L^2\}\hat{y}(t+h|t)=0, \tag{58}$$

which defines the forecast function, is

$$\hat{y}(t+h|t)=c_1+c_2\phi^h, \tag{59}$$

where c_1 and c_2 are constants which reflect the initial conditions. These constants are found by solving the equations

$$\begin{aligned} y_{t-1}&=c_1+c_2\phi^{-1},\\ y_t&=c_1+c_2. \end{aligned} \tag{60}$$

The solutions are

$$c_1=\frac{y_t-\phi y_{t-1}}{1-\phi}\qquad\text{and}\qquad c_2=\frac{\phi}{\phi-1}(y_t-y_{t-1}). \tag{61}$$

The long-term forecast is $\bar{y}=c_1$ which is the asymptote to which the forecasts tend as the lead period h increases.

The model of this example has a straightforward physical analogy. One can imagine a particle moving in a viscous medium under the impacts of its molecules which are in constant motion. The velocity $v(t)$ of the particle is governed by the equation $(I-\phi L)v(t)=\varepsilon(t)$, where $\phi\in[0,1)$ is a factor which reflects the viscosity of the medium and which governs the decay of the particle's velocity. The equation $(I-L)y(t)=v(t)$, which is equation (56) in another form, gives the position of the particle. The forecast function reflects the fact that, if the impacts which drive the particle through the medium were to cease, then it would come to rest at a point. Figure 2 represents these circumstances.

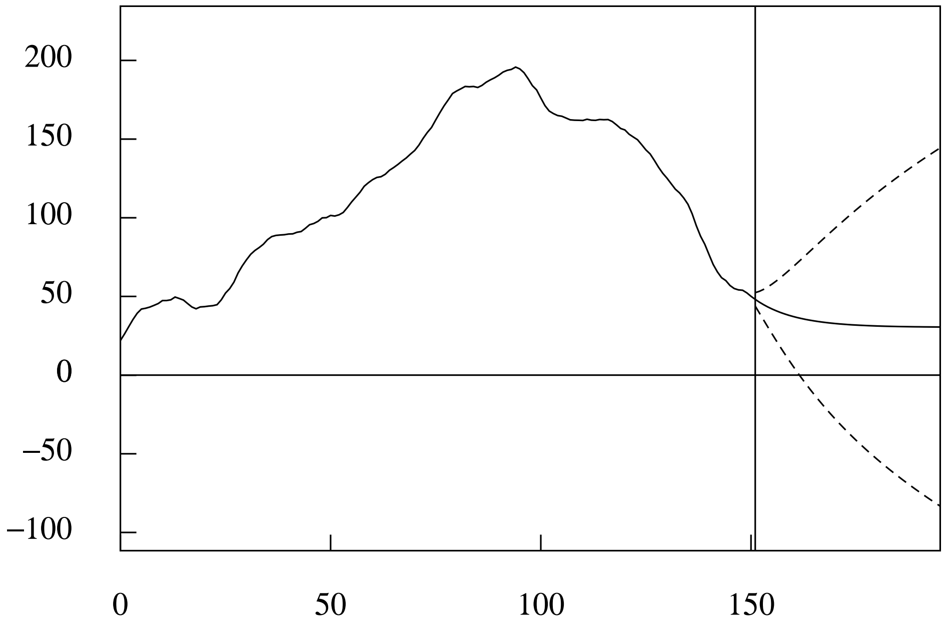

Figure 2. The graph of 150 observations on a simulated series generated by the AR(2) process $(1 - 1.9L + 0.9L^2)y(t) = \varepsilon(t)$ followed by 45 forecast values.