KINEMATICS AND DYNAMICS

Kinematics.

Imagine a body which is travelling in a straight line with a uniform acceleration of a. Let v_0 be the velocity at time $t = 0$ and let v be the velocity after t seconds. Then

$$v = v_0 + at. \tag{1}$$

Let x be the distance travelled in the interval of time. Then

$$x = \frac{(v_0 + v)}{2}t, \tag{2}$$

since $(v + v_0)/2$ represents the average velocity. Substituting in (2) the expression for v under (1) gives

$$x = \frac{(v_0 + v_0 + at)}{2}t, \qquad \text{or} \qquad x = v_0 t + \frac{1}{2}at^2. \tag{3}$$

Equally, this is obtained by integrating (1) with respect to t.

Now rewrite (1) as

$$a = \frac{v - v_0}{t} \tag{4}$$

Combining this with (2) gives

$$\begin{aligned} ax &= \left(\frac{v - v_0}{t}\right)\left(\frac{v_0 + v}{2}\right)t = \frac{v^2 - v_0^2}{2}, \\ &\text{or, equivalently,} \qquad v^2 = v_0^2 + 2ax. \end{aligned} \tag{5}$$

Dynamics

Let f be a force acting upon a body of mass m be a mass. Then Newton's Second Law of Motion asserts that the body is accelerated along a straight line in proportion to the magnitude of the force. Thus

$$f = ma \qquad \text{or} \qquad \text{force} = \text{mass} \times \text{acceleration}. \tag{5}$$

The unit of force is a newton which has the dimension of $k \times m \times s^{-2}$ in the mks (metres, kilograms, seconds) system.

Next we may define the primitive concept of momentum which is the product of mass and velocity. Then force can be represented as the rate of change of momentum:

$$\frac{d}{dt}mv = ma = f. \tag{7}$$

Also, from (1),

$$mv - mv_0 = ft \qquad \text{or} \qquad \text{impulse} = \text{force} \times \text{time}. \tag{8}$$

Another primitive concept is work which is force times the distance along its line of action:

$$w = fx. \tag{9}$$

The unit of work in the *mks* system is s a joule. The cost of doing work is an expenditure of energy. Thus energy is the capacity for doing work. Kinetic energy is energy due to motion. Potential energy is energy due to position.

Imagine that a body starts at rest and is accelerated uniformly to velocity of v in the time t. The uniform acceleration is $a = v/t$. The distance travelled is $x = vt/2$. The force by which this is achieved is $f = ma = mv/t$; and therefore the product of the force and the distance is

$$w = \frac{1}{2}mv^2. \tag{10}$$

Also, it follows from equation (5) that

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + fx, \tag{11}$$

since $f = ma$.